Shape analysis on Lie groups and Homogeneous Manifolds with applications in computer animation

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Geometric Numerical Integration

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Outline

- Analysis of shapes in vector spaces.
- Character animation and skeletal animation.
- Analysis of shapes on Lie groups.
- Applications in:
    - curve blending,
    - projection from open curves to closed curves, and
    - distances between curves.
- Examples in computer animation.

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Image recognition: a tennis player a tennis racket and a ball.

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Definition of **shapes** via an equivalence relation: let $\mathrm{I} \subset \mathbb{R}$ an interval, consider

$$\mathcal{P} := \mathrm{Imm}(\mathrm{I}, \mathcal{M}) = \{c \in C^{\infty}(\mathrm{I}, \mathcal{M}) \mid \dot{c}(t) \neq 0\},$$

$\mathcal{P}$ is called **pre-shape space** (an infinite dimensional manifold). Let $c_0, c_1 \in \mathcal{P}$ then

$$c_0 \sim c_1 \Longleftrightarrow \exists\, \varphi : \quad c_0 = c_1 \circ \varphi$$

with $\varphi \in \mathrm{Diff}^{+}(\mathrm{I})$ a orientation preserving diffeomorphism on I

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Shape space:

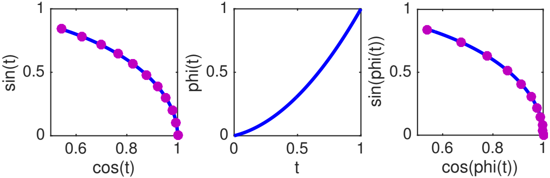
$$\mathcal{S} := \mathrm{Imm}(\mathrm{I}, \mathcal{M})/\sim$$

Let

$$\mathrm{Diff}^{+}(\mathrm{I}) = \{\varphi \in C^{\infty}(\mathrm{I},\mathrm{I}) \,|\, \varphi'(t) > 0\}$$

be the set of smooth, orientation preserving, invertible maps.

Open curves. $\mathrm{I} = [0,1]$



$(\cos(t), \sin(t))$, $\quad \varphi : t \mapsto \frac{3}{4}t^2 + \frac{t}{4}$, $\quad (\cos(\varphi(t)), \sin(\varphi(t)))$

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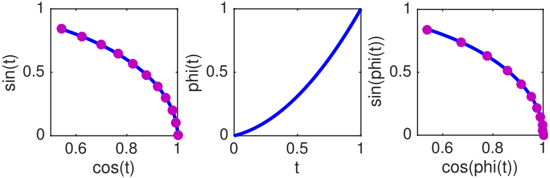

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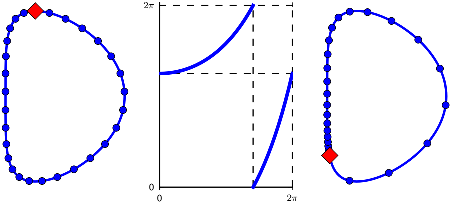
Closed curves. $\mathrm{Diff}^+(S^1) = \{\varphi \in C^\infty(S^1, S^1) \,|\, \varphi'(t) > 0\}$



Applications often require a **distance** function to measure similarities between shapes. Let $d_{\mathcal{P}}$ be a distance function on $\mathcal{P}$ (on parametrized curves) then

Distance on $\mathcal{S}$:

$$d_{\mathcal{S}}([c_0],[c_1]) := \inf_{\varphi \in \mathrm{Diff}^+(I)} d_{\mathcal{P}}(c_0, c_1 \circ \varphi).$$

But we need also to check that $d_{\mathcal{S}}$ is well defined, i.e. is independent on the choice of representatives c_0 for $[c_0]$ and c_1 for $[c_1]$.

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Computational methods for $d_{\mathcal{S}}$:

- gradient flows
- dynamic programming

Distance on $\mathcal{P}$

$d_{\mathcal{P}}(c_0, c_1)$ it is usually obtained by defining a Riemannian metric on the infinite dimensional manifold $\mathcal{P}$ of parametrized curves:

$$\mathcal{G}_c : T_c\mathcal{P} \times T_c\mathcal{P} \to \mathbb{R}$$

and taking the length of the geodesic α between $c_0 \in \mathcal{P}$ and $c_1 \in \mathcal{P}$ wrt $\mathcal{G}_c$, i.e.

$$d_{\mathcal{P}}(c_0, c_1) := \mathrm{length}(\alpha(c_0, c_1))$$

$\alpha(c_0, c_1)$ is the shortest path between c_0 and c_1 wrt $\mathcal{G}_c$

We will proceed differently and use the SRV transform

$$\mathcal{R} : \mathcal{P} \to \mathcal{C},$$

with $\mathcal{C}$ a vector space where we can (and will) use the L_2 metric.

- **Virtual character**: a closed surface in $\mathbb{R}^3$ (triangle mesh).
- **Motions** of characters via *skeletal animation* approach.
- **Skeleton** consisting of bones connected by joints.

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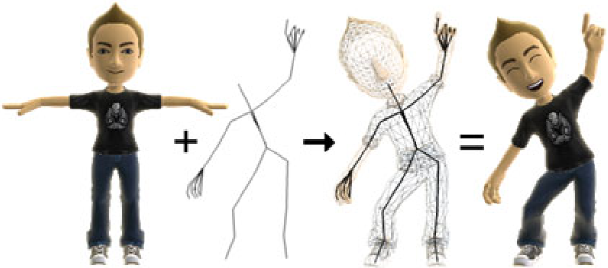

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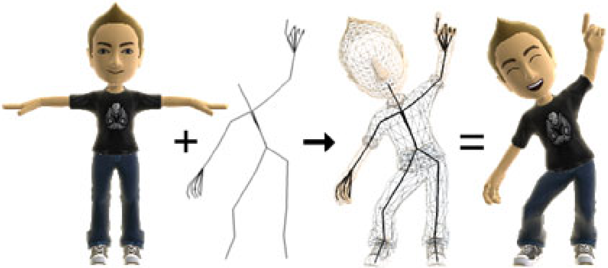

The **coordinates of the vertices** of the triangle mesh are specified in a coordinate system **aligned with the bone**. In the animation the movement of the vertices is determined by the bones.

- Skeleton: rooted tree made of bones and joints.
- Configuration space $\mathcal{J} = SE(3)^n$ or $\mathcal{J} = SO(3)^n$ (for human caracters).
- Character's pose specified by assigning values to the degrees of freedom.
- Animation: $\alpha : [a, b] \rightarrow \mathcal{J}$, a curve on $\mathcal{J}$, $[a, b]$ interval of time.
- *Motion capturing* (recording curves on $\mathcal{J}$).
- *Motion manipulation* consider entire animations as shapes belonging to $\mathcal{S}$.

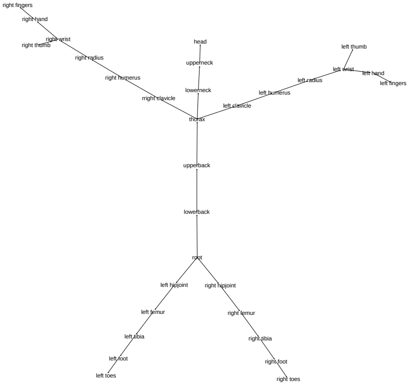

Figure: Skeleton: each node corresponds to a $Q(t) \in SO(3)$

Generating the data: $\alpha : [a, b] \to \mathcal{J}$, a curve on $\mathcal{J} = SO(3)^n$

motion capturing with and without markers

$\mathcal{P} := \mathrm{Imm}(\mathrm{I}, G)$	smooth functions with first derivative$\neq 0$
$\mathcal{S} := \mathrm{Imm}(\mathrm{I}, G)/\mathrm{Diff}^{+}(\mathrm{I})$	shape space

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Plan: we aim to obtain a **distance** function on $\mathcal{S}$ by

$$d_{\mathcal{S}}([c_0],[c_1]) := \inf_{\varphi \in \mathrm{Diff}^+(\mathrm{I})} d_{\mathcal{P}}(c_0, c_1 \circ \varphi) \tag{1}$$

with $d_{\mathcal{P}}$ a distance on $\mathcal{P}$.

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Definition. We say $d_{\mathcal{P}}$ is a reparametrization invariant distance function on $\mathcal{P}$ iff

$$d_{\mathcal{P}}(c_0, c_1) = d_{\mathcal{P}}(c_0 \circ \varphi, c_1 \circ \varphi) \quad \forall \varphi \in \mathrm{Diff}^+(\mathrm{I}). \tag{2}$$

Proposition

If $d_{\mathcal{P}}$ is a reparametrization invariant distance function on $\mathcal{P}$ then $d_{\mathcal{S}}([c_0],[c_1])$ as defined in (1) is independent of the choice of representatives of $[c_0]$ and $[c_1]$.

Tools to work with curves on G

$C_*^\infty(I, G)$ is an infinite dimensional Lie group, $C^\infty(I, \mathfrak{g})$ infinite dimensional Lie algebra

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The **evolution operator** of a regular (e.g. finite dimensional) Lie group G with Lie algebra $\mathfrak{g}$:

$$\mathrm{Evol}: C^\infty(\mathrm{I},\mathfrak{g}) \to C^\infty_*(I,G) := \{c \in C^\infty(\mathrm{I},G) \,:\, c(0)=e\}$$

$$\mathrm{Evol}(q)(t) := c(t), \quad \text{where} \quad \frac{\partial c}{\partial t} = \mathrm{R}_{c(t)*}(q(t)), \quad c(0)=e,$$

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$$\mathrm{Evol}: C^\infty(I,\mathfrak{g}) \to C^\infty_*(I,G)$$

is a **diffeomorphism**.
The inverse of the evolution operator is the so called *right logarithmic derivative*

$$\delta^r : C^\infty_*(I,G) \to C^\infty(I,\mathfrak{g}),$$
$$\delta^r g := \mathrm{R}^{-1}_{g*}(\dot{g}).$$

H. Glöckner, arXiv:1502.05795v3, March 2015.

A. Kriegl and P. W. Michor, The convenient setting of global analysis.

SRVT: square root velocity transform. Let $\langle\cdot,\cdot\rangle$ be a right-invariant metric on G and $\|\cdot\|$ the induced norm on tangent spaces,

$$\mathcal{R}:\{c\in \mathrm{Imm}(\mathrm{I},G)\,|\,c(0)=e\}\to\{v\in C^\infty(\mathrm{I},\mathfrak{g})\,|\,\|v(t)\|\neq 0\}$$

$$q(t)=\mathcal{R}(c)(t):=\frac{\delta^r c}{\sqrt{\|\delta^r c\|}}=\frac{\mathrm{R}^{-1}_{c(t)*}(\dot{c}(t))}{\sqrt{\|\dot{c}(t)\|}}$$

with inverse

$\mathcal{R}^{-1}:\{v\in C^\infty(\mathrm{I},\mathfrak{g})\,|\,\|v(t)\|\neq 0\}\to\{c\in \mathrm{Imm}(\mathrm{I},G)\,|\,c(0)=e\}$,

$\mathcal{R}^{-1}(q)(t)=\mathrm{Evol}(q\|q\|)=c(t)$.

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Reparametrization invariant distance on $\mathcal{P}$:

$$d_{\mathcal{P}}(c_0,c_1):=d_{L^2}(\mathcal{R}(c_0),\mathcal{R}(c_1))=\left(\int_{\mathrm{I}}\|q_0(t)-q_1(t)\|^2dt\right)^{\frac{1}{2}}$$

Proposition

$d_{\mathcal{P}}$ is reparametrization invariant.

Distance on $\mathcal{S}$

$$d_{\mathcal{S}}([c_0],[c_1]):=\inf_{\varphi\in\mathrm{Diff}^+(\mathrm{I})}\left(\int_{\mathrm{I}}\|q_0(t)-q_1(t)\|^2dt\right)^{\frac{1}{2}}$$

Elastic metric on $\mathcal{P} := \mathrm{Imm}(\mathrm{I}, G)$. Using the right invariant metric $\langle\cdot,\cdot\rangle$ on G we can define

$$\mathcal{G} : T\mathcal{P} \times T\mathcal{P} \mapsto \mathbf{R}$$

where

$$\begin{aligned} \mathcal{G}_c(h,k): &= \int_{\mathrm{I}} \left[a^2 \langle D_s h, v\rangle \langle D_s k, v\rangle \right] ds \quad \text{(tangential)} \\ \text{(normal)} &+ \int_{\mathrm{I}} \left[b^2 \left(|D_s h - \langle D_s h, v\rangle v|^2 \, |D_s k - \langle D_s k, v\rangle v|^2 \right) \right] ds, \end{aligned}$$

here the integration is with respect to arc-length, $ds = |c'(t)|dt$, and

$$v := \frac{c'(t)}{|c'(t)|}, \qquad D_s h := \frac{\partial h(t(s))}{\partial s} = \frac{1}{|c'(t)|}\frac{\partial h}{\partial t}.$$

A family of Sobolev type metrics of order one.

Assume $\mathrm{I} = [0,1]$, $\mathcal{R} : \mathrm{Imm}(\mathrm{I}, G) \to C^\infty(\mathrm{I}, \mathfrak{g})$

Theorem

The pullback of the L_2 inner product on $C^\infty(\mathrm{I}, \mathfrak{g})$ to $\mathcal{P} = \mathrm{Imm}(\mathrm{I}, G)$, by the SRV transform $\mathcal{R}$ is the elastic metric

$$G_c(h,h) = \int_I \left| D_s h - \langle D_s h, v\rangle v \right|^2 + \frac{1}{4} \langle D_s h, v\rangle^2 \, ds,$$

and

$$D_s h = \frac{\dot{h}}{\|\dot{c}\|}, \qquad v = \frac{\dot{c}}{\|\dot{c}\|}.$$

D_s denotes differentiation with respect to arc length, v is the unit tangent vector of c and ds denotes integration with respect to arc length.

For computational purposes, we can transform the curves with $\mathcal{R}$ and then use the L_2 metric.

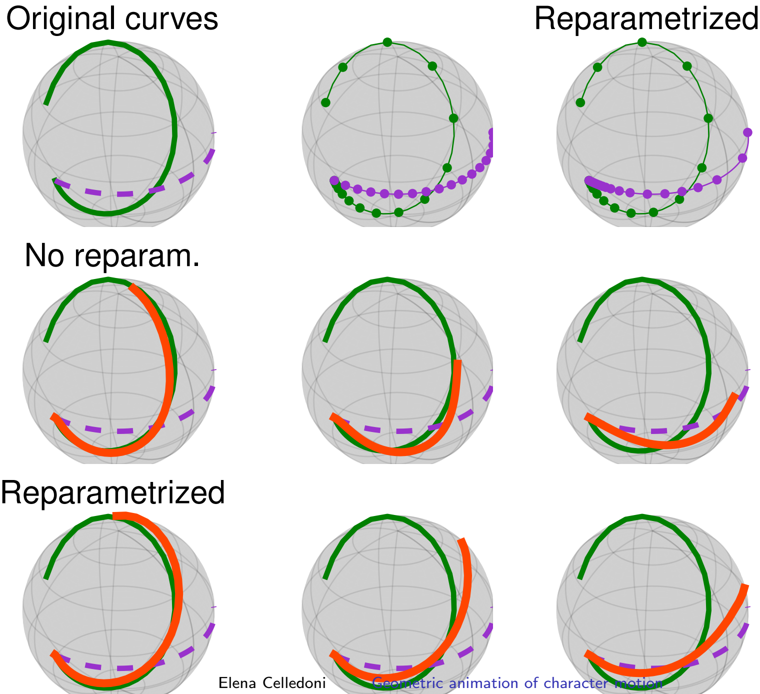
Motion blending on G

Geodesic paths $\alpha : [0, 1] \to \mathcal{P}$ between two parametrized curves on the Lie group G:

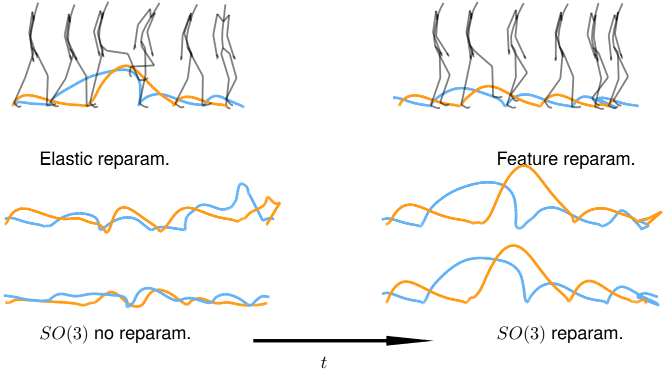
$$c = \alpha(0, t), \quad d = \alpha(1, t)$$

we get (**visually convincing**) deformations from c to d

$$\alpha(x, t) = \mathcal{R}^{-1}((1 - x)\mathcal{R}(c) + x\,\mathcal{R}(d))(t).$$


Original curves
Reparametrized
No reparam.
Reparametrized

Obstacle example (motion blending)



Using SRVT $\mathcal{R}$, we can identify curves $c \in \mathcal{P} = \mathrm{Imm}(I, G)$ with $\mathcal{R}(c) \in C^\infty(\mathrm{I}, \mathfrak{g})$

Open curves

$$\mathcal{C}^o := \mathcal{R}\big(\mathrm{Imm}(I, G)\big)$$

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Open curves

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Closed curves

$$\mathcal{C}^c := \mathcal{R}\big(\{c \in \mathrm{Imm}(I, G) \mid c(0) = c(1) = e\}\big)$$

$$\mathcal{C}^c = r^{-1}(e), \qquad r := \mathrm{ev}_1 \circ \mathrm{Evol} \circ \mathrm{sc}, \quad r : \mathcal{C}^o \to G$$

ev_1 is the evaluation operator evaluating a curve at $t = 1$, while sc is the map $\mathrm{sc}(q) := q\,\|q\|$, and we notice that $r(q) = \mathcal{R}^{-1}(q)(1)$.

Theorem

$\mathcal{C}^c$ is a submanifold of finite codimension of $C^\infty(I, \mathfrak{g})$

Projection from $\mathcal{C}^o$ onto $\mathcal{C}^c$ by means of a constrained minimization problem

$$\min_{q\in\mathcal{C}^c} \frac{1}{2}\|q-q_0\|, \qquad q_0 \in \mathcal{C}^o$$

Instead of minimizing the distance from closed curves to q_0 we minimize the closure constraint:

Projection from $\mathcal{C}^o$ onto $\mathcal{C}^c$ by means of a constrained minimization problem

$$\min_{q \in \mathcal{C}^c} \frac{1}{2} \| q - q_0 \|, \qquad q_0 \in \mathcal{C}^o$$

Instead of minimizing the distance from closed curves to q_0 we minimize the closure constraint:

Measuring closedness.
Consider $\phi : \mathcal{C}^o \to \mathbf{R}$

$$\phi(q) := \frac{1}{2} \| \log(r(q)) \|^2, \quad r := \mathrm{ev}_1 \circ \mathrm{Evol} \circ \mathrm{sc}$$

and

$$\phi(q) = 0 \Longleftrightarrow q \in \mathcal{C}^c$$

$$T_q\phi(f) = \langle \mathrm{grad}(\phi)(q), f\rangle_{L_2} = \int_I \langle \mathrm{grad}(\phi)(q), f\rangle\, dx,$$

Theorem

The gradient vector field wrt the L_2 inner product is

$$\mathrm{grad}(\Phi)(q) = \|q\|\ \alpha(q) + \langle \alpha(q), \frac{q}{\|q\|}\rangle\ q, \tag{3}$$

where

$$\begin{aligned}
\alpha(q) &:= \mathrm{Ad}^T_{c(q)^{-1}}\,\mathrm{Ad}^T_{r(q)}\ (\log(r(q))) \quad \in C^\infty(I,\mathfrak{g})\\
c(q) &:= \mathcal{R}^{-1}(q) \in C^\infty(I,G)\\
r(q) &:= \mathcal{R}^{-1}(q)(1) \in G.
\end{aligned}$$

Projection form $\mathcal{C}^o$ to $\mathcal{C}^c$:
Gradient flow

$$\frac{\partial u}{\partial \tau} = -\mathrm{grad}(\phi)(u), \quad u(t,0) = q(t).$$

$\bar{c}$ based on discrete points $\{\bar{c}_i := c(\theta_i)\}_{i=0}^{n}$

$$\bar{c}(t) := \sum_{k=0}^{n-1} \chi_{[\theta_k,\theta_{k+1})}(t)\ \exp\left(\frac{t-\theta_k}{\theta_{k+1}-\theta_k}\log(\bar{c}_{k+1}\bar{c}_k^{-1})\right)\bar{c}_k, \qquad (4)$$

where χ is the characteristic function.

SRV transform: $\bar{q} = \mathcal{R}(\bar{c})$ piecewise constant function in $\mathfrak{g}$:
$\bar{q} = \{\bar{q}_i\}_{i=0}^{n-1}$

$$\bar{q}_i := \frac{\eta_i}{\sqrt{\|\eta_i\|}}, \quad \eta_i := \frac{\log(\bar{c}_{i+1}\bar{c}_i^{-1}),}{\theta_{i+1}-\theta_i}.$$

The **inverse SRV** transform: $\bar{c} = \mathcal{R}^{-1}(\bar{q})$:

$$\bar{c}_{i+1} = \exp\left(\bar{q}_i\|\bar{q}_i\|\,(\theta_{i+1}-\theta_i)\right)\cdot\bar{c}_i$$

Curve reparametrization
Applying a reparametrization $\varphi \in \mathrm{Diff}(\mathrm{I})$ to the discrete curve $\bar{c}$ gives $\tilde{c}$ with $\{\tilde{c}_i\}_{i=0}^n$:

$$\tilde{c}_i := \bar{c}_j \exp(s \log(\bar{c}_{j+1}\bar{c}_j^{-1})), \qquad s := \frac{\varphi(\theta_i) - \theta_j}{\theta_{j+1} - \theta_j}, \qquad i = 0, \ldots, n,$$

where j is an index such that $\theta_j \leq \varphi(\theta_i) < \theta_{j+1}$. Note that $\tilde{c}_0 = \bar{c}_0$ and $\tilde{c}_n = \bar{c}_n$

Curve interpolation

$$\begin{aligned} &[0,1] \times \mathcal{P} \times \mathcal{P} \to \mathcal{P} \\ &(s, \bar{c}_0, \bar{c}_1) \mapsto \mathcal{R}^{-1}((1-s)\mathcal{R}(\bar{c}_0) + s\mathcal{R}(\bar{c}_1)), \end{aligned} \tag{5}$$

with interpolation parameter s.

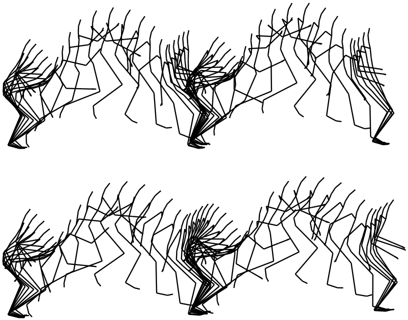
Curve closing in $SO(3)$

$$\mathrm{grad}(\phi)(q) = \|q\| c \log(c(1)) c^T + \langle c \log(c(1)) c^T, \frac{q}{\|q\|}\rangle q, \quad c = \mathcal{R}^{-1}(q)$$

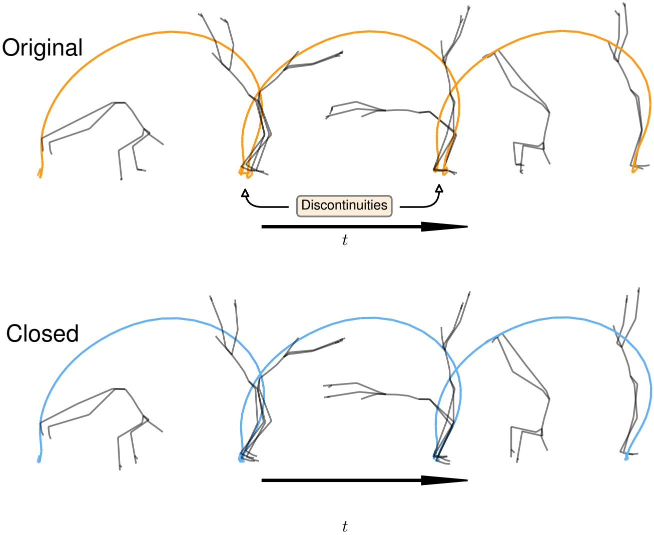
$$\bar{u}^{k+1} = \bar{u}^k - \alpha_k \,\mathrm{grad}(\phi)(\bar{u}^k),$$

where every $\bar{u}^k$ is a discrete curve as defined above, i.e., $\bar{u}^k = \{\bar{u}_i^k\}_{i=0}^n$.

Jumping example (closure of curves)



Handspring example (closure of curves)



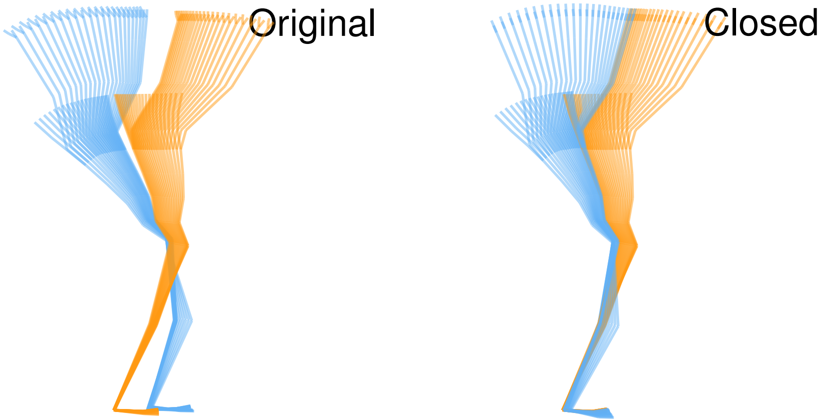


Figure: Discontinuities in the handspring animation.

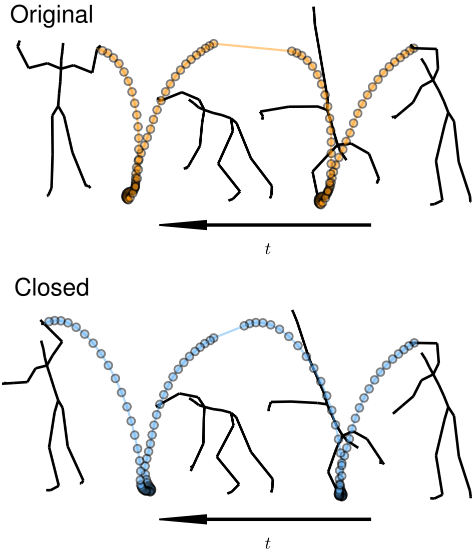


Figure: Application of closing algorithm to a cartwheel animation.

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Thank you for listening.