Recent research

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MFO, March 24, 2011

Poisson integrators for Lie-Poisson structures on $\mathbb{R}^3$

Poisson system

$$\dot{z} = B(z)\nabla H(z).$$

The classification of Lie-Poisson structures on $\mathbb{R}^3$,...
For a constant Poisson system, ...
When a constant matrix

$$B = \begin{pmatrix} 0_{m_1} & M \\ -M^T & 0_{m_2} \end{pmatrix},$$

where M is an $m_1 \times m_2$ constant matrix and $\mathrm{rank}(M) = r$, ...

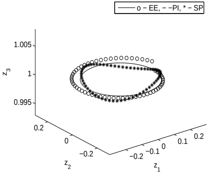
- Find a transformation $u = \phi(z) = (\phi_1(z), \phi_2(z), \phi_3(z))^T$, such that

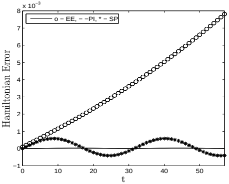
$$\phi_z(z)B(z)\phi_z(z)^T = \overline{B} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

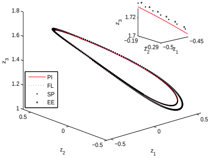
  Then we get $\dot{\phi(z)} = \overline{B}\nabla_{\phi(z)} h(\phi(z)), \quad h(u) = H(\lambda(u)), \ \lambda = \phi^{-1}.$
- SPRK methods are the Poisson integrators of this system (Local).
- Numerical experiments for two Lie-Poisson systems and compare our Poisson integrators with other known methods.

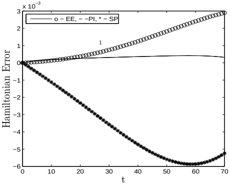
Poisson integrators for Lie-Poisson structures on $\mathbb{R}^3$

Our Poisson integrators (PI) with explicit Euler method (EE) and a standard splitting method (SP), exact flows are denoted by FL.









When can a SPRK method preserve the types of equilibrium points of Hamiltonian systems?

The dynamical behaviors of the numerical integrators is governed by the geometric properties. This is one of the reasons it is important to preserve the types of equilibrium points. The dynamics of a system in the neighborhood of an equilibrium point is generally decided by the eigenvalues of the linearized system at the equilibrium point.

Types: elliptic, hyperbolic, elliptic-hyperbolic.

- Normal forms of linear Hamiltonian systems
- For any step size AVF/Midpoint method can preserve...
- What about SRK and SPRK? The region of step size may not be the whole real line, then what it will be...

Nonlinear stability analysis of symplectic integrators

Numerical KAM theory, Nekhoroshev stability, based on modified Hamiltonian...

- Consider Hamiltonian systems with a homoclinic orbit, Hamiltonian function is

$$H = \frac{1}{2}p^2 + V(q), \quad (q, p) \in \mathbb{R}^2, \quad V \in C^\infty(\mathbb{R}).$$

  Any truncated modified Hamiltonian $\tilde{H}$ may have more critical points than H. With the step size h increasing, we prove how these additional critical points move, and we give the theorem how to get the critical h^*, when the homoclinic orbit disappears.
- Choose Test equation and define the nonlinear stability set—$(-h^*, h^*)$.
- Relationships between linear and nonlinear stability.