Two slides on Lie splitting for linear PDEs

Katharina Schratz
joint work with Alexander Ostermann

University of Innsbruck

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Parabolic partial differential equation

$$\partial_t w = \mathcal{L} w + \psi \qquad \text{in rectangle } \Omega$$

$$\mathcal{L}\,\cdot\; = \partial_x\big(p\partial_x \cdot\big) + \partial_y\big(q\partial_y \cdot\big) \qquad \text{strongly elliptic}$$

with i.c. and hom. Dirichlet b.c.

Formulation as an abstract evolution equation in $L^2(\Omega)$

$$u'(t) = Lu(t) + g(t) = (A + B)u(t) + g(t), u(0) = u_0\,,$$

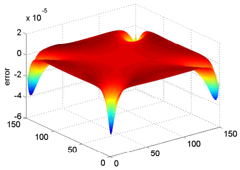
where $u(t) = w(t, \cdot, \cdot)$ and $g(t) = \psi(t, \cdot, \cdot)$.

Theorem: resolvent Lie splitting

$$u_{n+1} = (I - hB)^{-1}(I - hA)^{-1}(u_n + hg(t_n)).$$

is convergent of order γ, $0 \le \gamma \le 1$, if $g(t) \in \mathscr{D}(A^{\gamma} B L^{-1})$.

Q: Conditions on $g(t)$ s.t. $g(t) \in \mathscr{D}(A^\gamma B L^{-1})$?



Problem: Ω rectangle $\longrightarrow$ corner singularities

$\gamma < \frac{1}{4}$: $g(t) \in H^{\frac{1}{2}}(\Omega)$

$\gamma = 1$: $g(t) \in H^2(\Omega)$ and $\psi(t,\cdot)|_{\text{corners}} = 0$

Modified Lie splitting: (Douglas-Rachford method)

$$u_{n+1} = (I - hB)^{-1}(I - hA)^{-1}\Big(\big(I + h^2 AB\big)u_n + hg(t_n)\Big),$$

Theorem: Convergence order 1 for $g(t) \in L^2(\Omega)$!

Exponential Lie splitting in $L^p(\Omega)$ spaces

$$\|e^{hA}\|_{L^p(\Omega)} \le C_A, \quad \|e^{hB}\|_{L^p(\Omega)} \le C_B, \quad C_A, C_B > 1$$

Q: Is the exponential Lie splitting stable, i.e.

$$\Big\|\Big(e^{hA}e^{hB}\Big)^n\Big\|_{L^p(\Omega)} \le C \quad \text{Yes!}$$