# Multimodal Elastic Image Matching

Research results based on my diploma thesis supervised by Prof. Witsch[2] and in cooperation with Prof. Mai[3].

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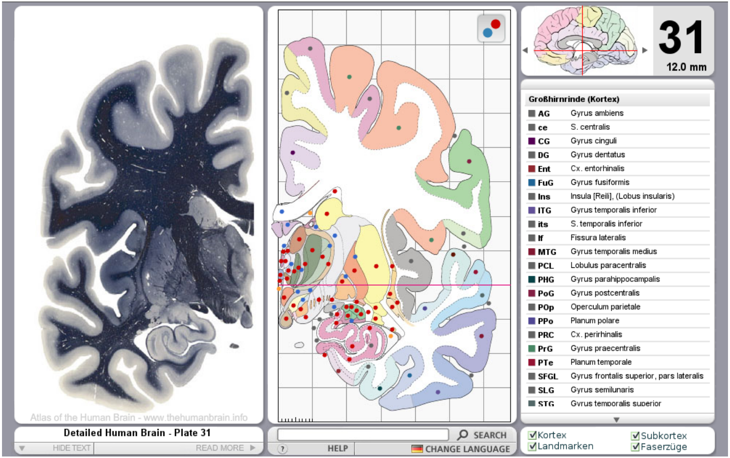
# Multimodal Elastic Image Matching—Abstract

In brain research one often deals with three dimensional (3d) magnet resonance images (MRI) of the individual human brain, from which one usually analyzes sectional views. The "Atlas of the human brain" by Prof. Mai et al. is a great tool as a reference to define the positions of different regions of the brain. Therefore, a matching of this atlas to the individual MRI is needed.
An exclusively rigid or linear transformation can be defined easily by a set of control points. Of course, this kind of transformation cannot ensure a global matching. A more versatile transformation is possible, but it requires a huge amount of time to set the large number of control points manually by viewing sequences of 2d slices of the 3d image. The quality of the transformation strongly depends on the selection of the control points. Therefore a fully automated matching process by a computer, which takes into account all three dimensions simultaneously, is desired.
In this talk the focus is on multimodal non-linear matching. Therefore we establish an appropriate definition of similarity between two images of different modality and introduce a model of elastic transformation to preserve coherence. Methods to stabilize the matching process, to circumvent minor drawbacks and to speed up computational time are also presented.
Although the topic is of strong mathematical background, this talk will be descriptive and aims at a non-mathematical audience.

# The „Atlas of the Human Brain“ (J.K. Mai et al.)




www.thehumanbrain.info/brain/bn_brain_atlas/brain.html

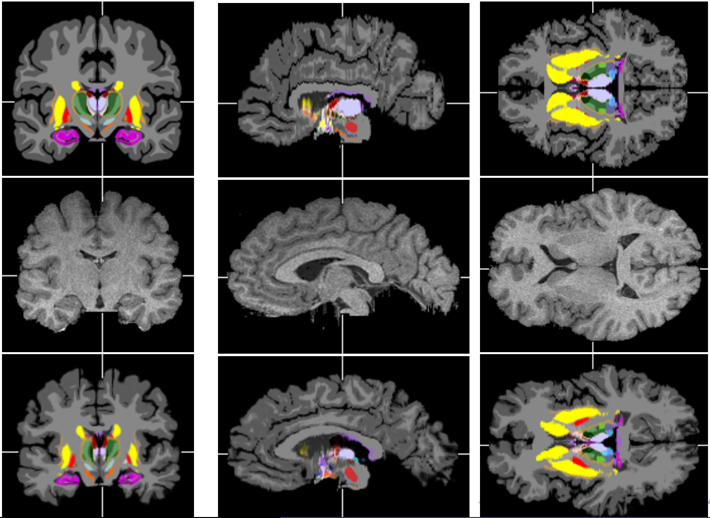
# Our task

coronal sagittal horizontal

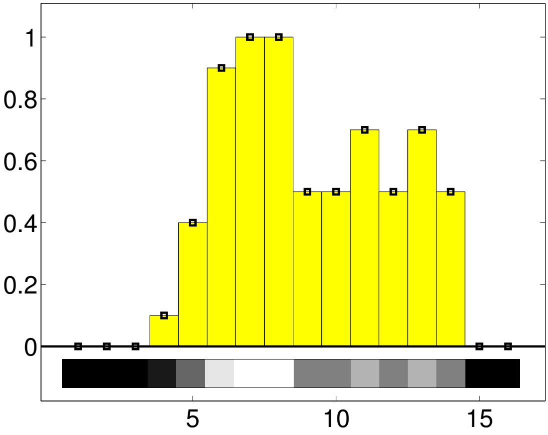
Atlas

MRI

matched Atlas



# Definition of an Image




2d image (foto)

- pixel (picture element)

3d image (MRI)

- voxel (volume element)

gray level image

- one value per pixel/voxel

color picture

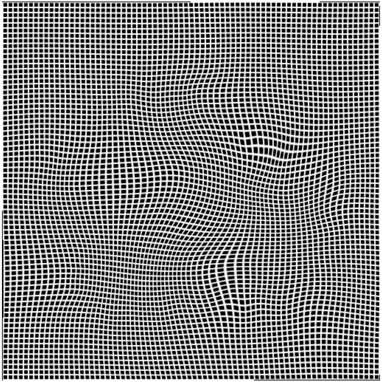
- three values per pixel/voxel

# 2d example images

reference image $R$

template image $T$

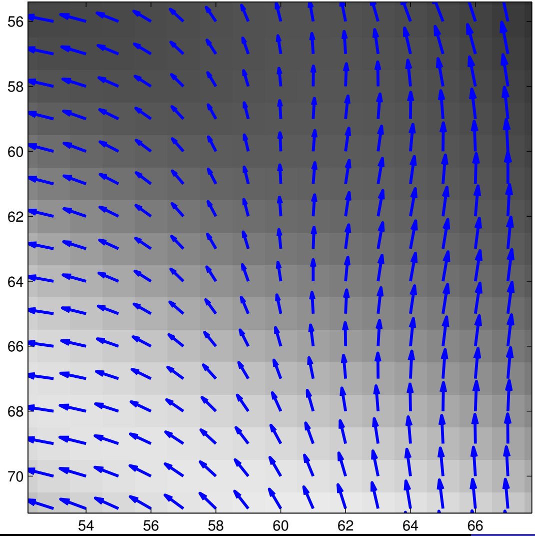
# Wanted



coordinate transformation
$y(\vec{u}, \vec{x}) := \vec{x} - \vec{u}(\vec{x})$, in short $y$

transformed template image
$T \circ y$

# The displacement field $\vec{u}$

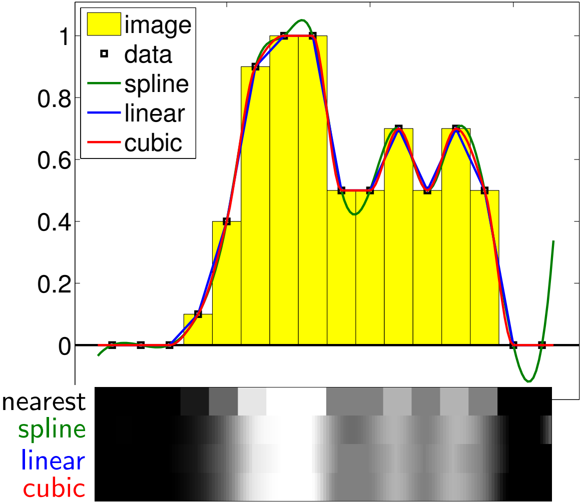



coordinate transformation

$$y(\vec{u}, \vec{x}) := \vec{x} - \vec{u}(\vec{x})$$

- arrows point to position in $T$, where color is picked up
- interpolation is required

# Interpolation




nearest neigbour (yellow bars)

+ easiest
− values jump

spline

+ extreme smooth
− expensive computation
− overshoots

linear

+ easy to compute
− intermediate values

cubic

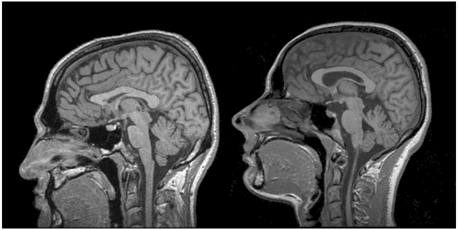
+ smooth
+ computation time ok

# Control points

- define set of (strong or weak) control points and map these points
  - strong control points: the transformation $y$ has to map these points
  - weak control points: these points shall lie close together in the matched images
- manual selection of control points: long-lasting and susceptible for inaccuracy
- possible are rigid transformations and affin-linear transformations
- these transformations do fit only locally

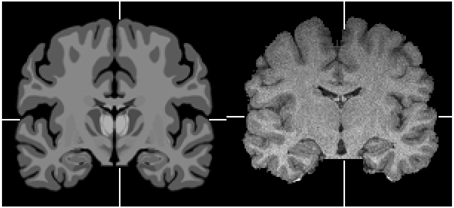
# Monomodal and Multimodal Matching

## Monomodal Matching



- corresponding regions ↔ similar gray levels
- try to fuse gray levels

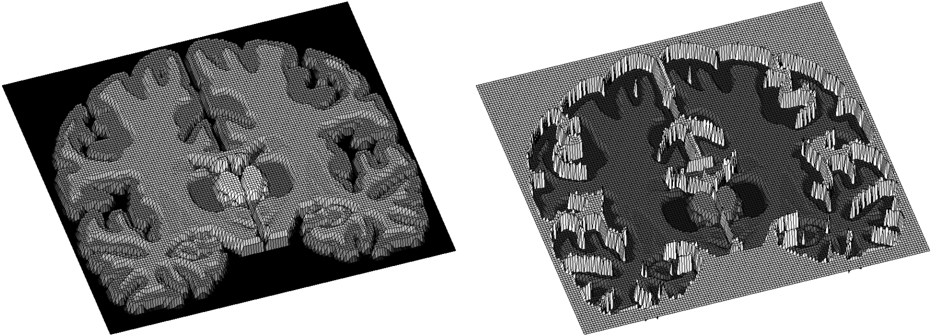
## Multimodal Matching



- corresponding regions ↔ arbitrary gray level
- corresponding feature: contour lines

# Contour lines

Surface plot of an atlas slice with two different assignments of gray levels



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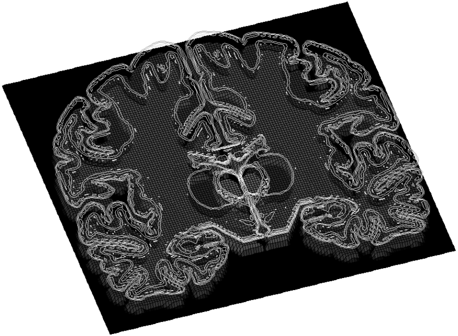
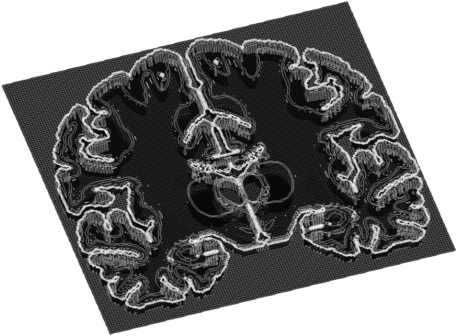
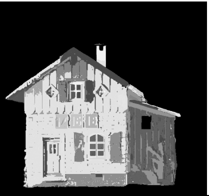




# Features of Similarity

- in general, the gray levels are different
- the contour lines correspond

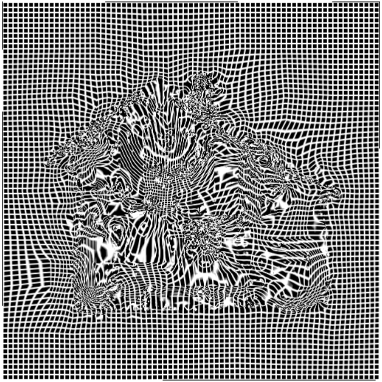


# The Optimization Problem

Find a transformation $y$, such that the contourlines coincide.

- consider crossing of contour lines
- sum this values over the whole picture domain
- determine a transformation $y$ such that this distance is minimal

# Result of Minimization



- the outer shape of $T \circ y$ and $R$ matches.
- the coherence is complety lost.
- this result is useless.

# Regularization

- preserve the coherence/topology

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  - the displacement field $\vec{u}$ of one voxel and the neighbouring voxel must be similar, i.e. $\vec{u}(\vec{x}) \approx \vec{u}(\vec{x} + \epsilon \cdot \vec{h})$ for small $\epsilon > 0$ and any direction $\vec{h}$.

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    - at least the slope of the displacement field $\vec{u}$ must be small.
    - maybe the curvature of the displacement field $\vec{u}$ must be small, too.
  - Introduce a penalty term
    - find a displacement field $\vec{u}$ that minimizes the distance of the images $R$ and $T$ $+ \alpha\cdot$ punishment by slope or curvature of $\vec{u}$.

# Advanced Minimization Problem

Find a displacement field $\vec{u}$ that minimizes the distance of the images $R$ and $T$ $+\ \alpha\cdot$ punishment by slope or curvature of $\vec{u}$.

$$\text{minimize}_{\vec{u}} \quad \underbrace{\mathcal{D}_{R,T}\big(y(\vec{u})\big) + \alpha \cdot S(\vec{u})}_{I_\alpha(\vec{u})}$$

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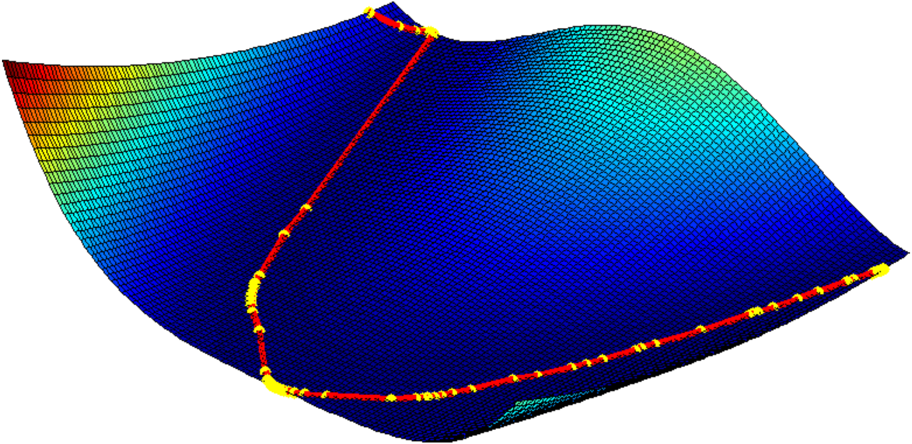
- We get a non-linear boundary value problem.

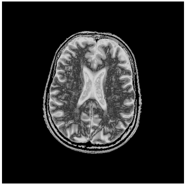
# Advanced Minimization Problem

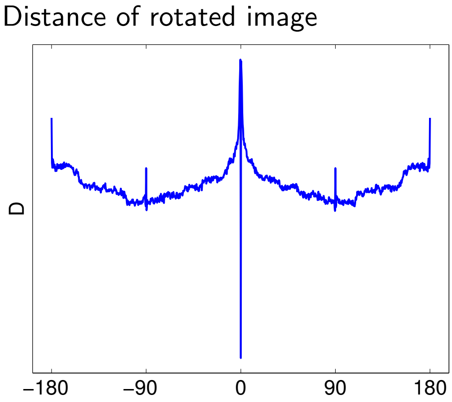
Going down the hill in case of two unknowns.



# Local Minima

0

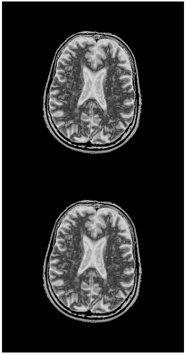


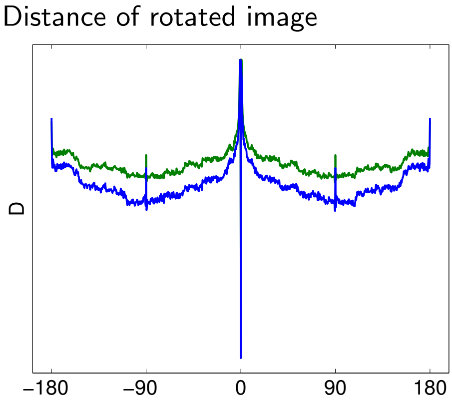


# Local Minima

0

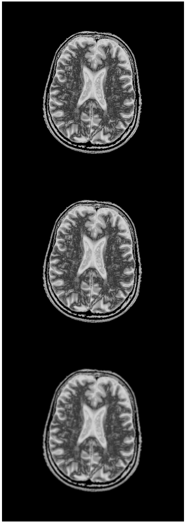
1

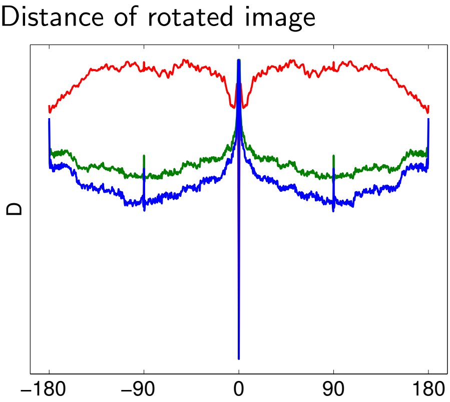




# Local Minima

0
1
2





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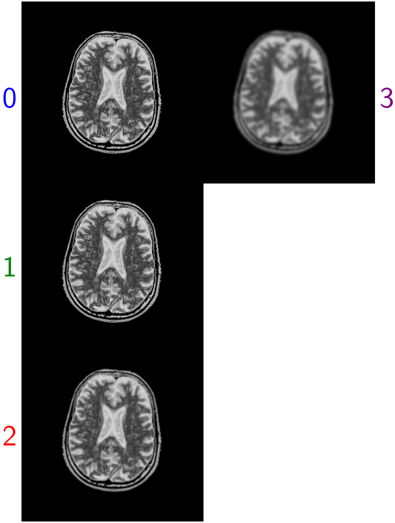


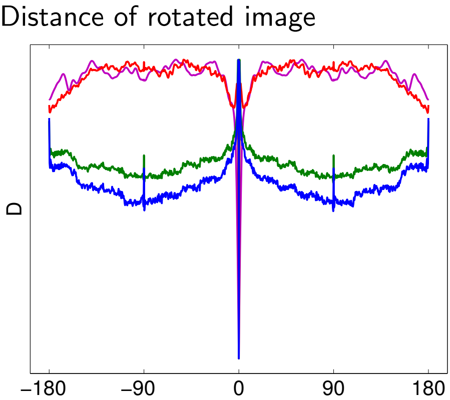

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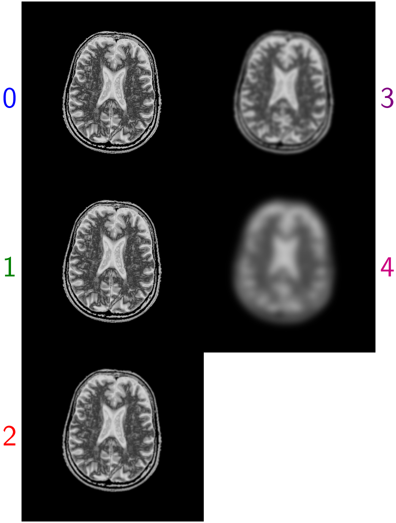
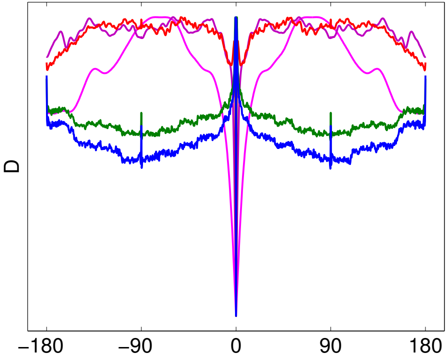



Distance of rotated image



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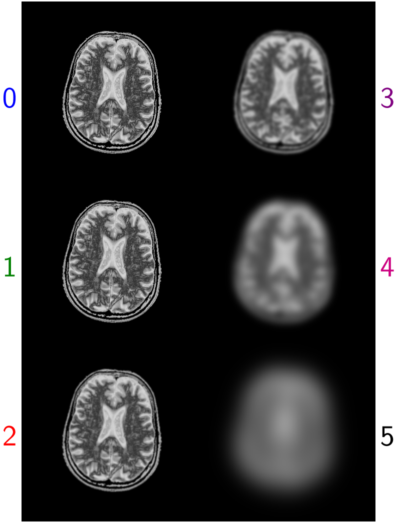
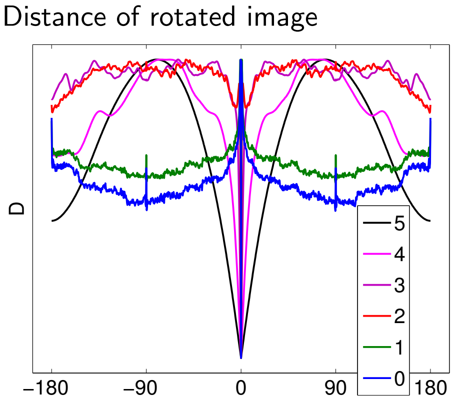


# Basic Algorithm

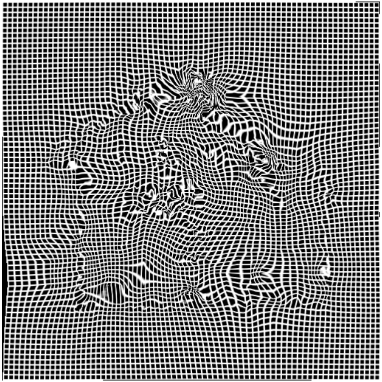
```
from smooth to sharp
    while images get more similar do
        compute direction of deformation
        apply this direction with optimal step size
    end while
end from
```

# Given images

Reference $R$

Template $T$

# Weak stiffness in the elastic transformation

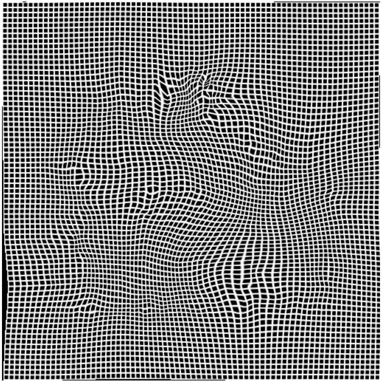


coordinate transformation

Templatebild $T \circ y_{\text{weak}}$

- Artificial deformations occur locally.
- The stiffness is to weak.

# Medium stiffness in the elastic transformation

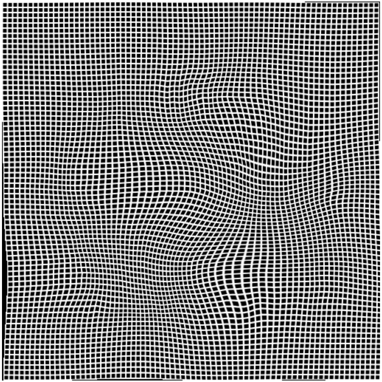


coordinate transformation

Template $T \circ y_{\text{medium}}$

- A few local artificial deformations occur.
- The quality is better but not satisfactory.

# Strong stiffness in the elastic transformation

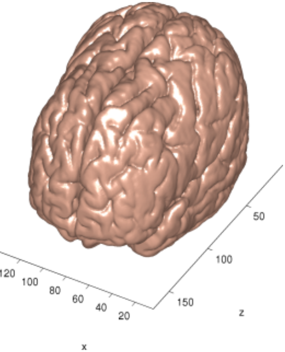


coordinate transformation

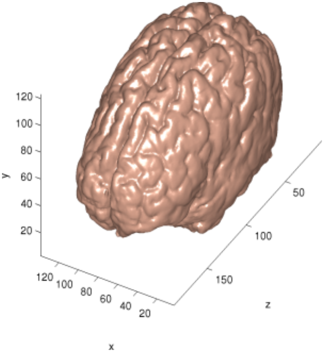

Template $T \circ y_{\text{strong}}$

- No artificial deformations.
- The strong stiffness is good, but it inhibits strong local deformations.

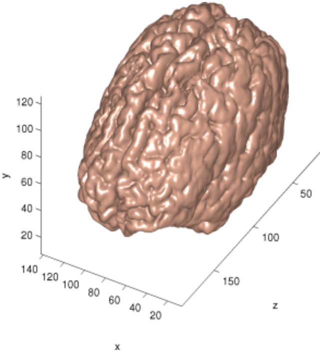
# Elastic Image Registration – Reconstruction of the Surface




Atlas brain




Patient brain




matched Atlas

# Summary

- Multimodal image matching relies on contour lines.
- The non-fitting of contour lines defines the distance of both images.
- The transformation minimizes the distance.
- The elastic model of transformation preserves the coherence.
- Smoothing prevents to be stuck in a local minima.

# For further information, see

http://na.math.kit.edu/loechel/research/imgreg/